Nucleon-Nucleon Scattering with Inelastic Unitarity*

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The effects of inelastic unitarity on dynamical calculation of nucleon-nucleon scattering are studied, using the N/D formalism for partial-wave amplitudes modified to take into account reaction channels by means of the inelastic factor η. The equations are solved for the ${}^1D_2$ amplitude for which the inelastic scattering is *known* (for laboratory kinetic energies $E_L \lesssim 800$ MeV) and *large*. We use left-hand-cut Born terms determined by Scotti and Wong. The calculated (real part of the) phase shift δ agrees very closely with the Scotti-Wong results for $E_L \lesssim 200$ (for reasonable choices of the high-energy behavior of η) but deviates appreciably at higher energy, peaking at $\approx$400–500 MeV and going negative at $\gtrsim$1 BeV.

I. INTRODUCTION

NUCLEON–NUCLEON scattering has been studied intensively both experimentally and theoretically. The experimental data yield a unique set of (real) phase shifts for incident laboratory kinetic energy $E_L \lesssim 300$ MeV.[1] In addition, there are nonunique (complex) phase-shift analyses at $\sim$650 and 970 MeV.[2,3] On the other hand, dynamical calculations have been mainly concerned with energies $E_L \lesssim 400$ MeV where inelastic nucleon-nucleon scattering is negligible (it is zero below 300 MeV). These calculations have had a good deal of success in fitting the phase shifts (or the observables directly) as a function of energy with a reasonably small number of parameters. For example, the dispersion theory calculations of Scotti and Wong (S–W),[4] who consider the multimeson exchanges between the nucleons to proceed via the recently discovered multipion resonances, have been very successful.

However, these dynamical calculations of the elastic scattering still depend on the scattering to inelastic channels even if, in the region of interest, no inelastic scattering is energetically possible. The inelastic channels are in practice usually ignored. In particular, solution of partial wave dispersion relations involves integrals over *all* physical energies (as well as the left-hand cut). S–W assumed that only elastic scattering occurs at all E_L.

The purpose of this article is to study the effects of including inelastic unitarity on the physical cut in solving the partial wave dispersion relations for N–N scattering[5,6] We have in mind two features:

(i) Investigate whether including inelastic processes modifies the agreement of the S–W calculations with experiment in the energy range $E_L \lesssim 300$ MeV.

(ii) Extend the theoretical calculations to an energy region where we know that the inelastic scattering is large.

There are two different approaches that can be used to include inelastic effects. The multichannel N/D formalism is applicable but it has the disadvantages that only two body channels can be treated properly, many unknown parameters enter in the calculations, and the numerical solution of the coupled integral equations is very time consuming even on the fastest computers; of course one calculates more observables, i.e., the individual reaction cross sections. The approach we will use is the following: assume some knowledge of the total partial wave reaction cross section $\propto (1-\eta^2)$ [where $\eta = \exp(-2\delta^I)$ with δ^I the imaginary part of the elastic-scattering phase shift] and solve the one channel N/D equations modified to take into account inelastic processes by means of the factor η. We use the method developed by Frye and Warnock[7] which seems a more straightforward extension of the pure elastic case, $\eta(E)=1$, than that given by Froissart.[8] The relevant formalism is presented in Sec. II.

We investigated only the ${}^1D_2$ partial wave, since η for this amplitude has been determined from experi-

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[1] H. Stapp, H. Noyes, and M. Moravcsik, *Proceeding of the 1962 Annual International Conference on High Energy Physics* (CERN, Geneva, 1962), p. 131, and 1960 Conference at Rochester, p. 128; P. Signell, Phys. Rev. **133**, B982 (1964); G. Breit, M. Hull, K. Lassila, and K. Pyatt, Phys. Rev. **120**, 2227 (1960).
[2] L. Azhgirey *et al.*, Phys. Letters **6**, 200 (1963).
[3] Y. Hama and N. Hoshizaki, Kyoto University Report RIFP-34, 1964 (unpublished).
[4] A. Scotti and D. Wong, Phys. Rev. Letters **20**, 142 (1963); Proceedings of the Athens Topical Conference, April 1963 (to be published).
[5] Preliminary results were presented by G. Shaw and A. Scotti, Bull. Am. Phys. Soc. **9**, 62 (1964).
[6] Inelastic effects in N–N scattering have recently been considered by Y. Leung, Bull. Am. Phys. Soc. **9**, 62 (1964); and M. MacGregor, Phys. Rev. Letters **12**, 403 (1964); also see, M. Moravcsik, UCRL-7778, 1964 (unpublished).
[7] G. Frye and R. Warnock, Phys. Rev. **130**, 478 (1963).
[8] M. Froissart, Nuovo Cimento **22**, 191 (1961).

ment[9] (see Section III) up to $E_L \approx 800$ MeV and $1-\eta^2$ is large. Various choices of the high-energy behavior of η were considered. The left-hand-cut Born terms were taken from the calculations of S–W.

The results of the calculations are presented in Sec. III. In brief, they can be summarized as follows: The calculated (real part of the) phase shift δ for the ${}^1D_2$ partial wave agrees very closely with the S–W results for $E_L \lesssim 200$ MeV (for reasonable choices of the high-energy behavior of η). However, it deviates appreciably at higher energy, peaking at $\sim$400–500 MeV and going negative at $\gtrsim 1$ BeV.[10]

In conclusion, we would like to stress that although it seems like a very good approximation to neglect inelastic unitarity for dynamical calculations in the energy range $E_L \lesssim 200$ MeV, any quantitative dynamical calculation for $E_L \gtrsim 300$ MeV must include the inelastic cut. It then becomes important to experimentally extend our knowledge of the inelastic cross sections to higher energy.

II. N/D EQUATIONS WITH INELASTIC UNITARITY

Since our calculations will be restricted to the singlet amplitude, consider the scattering of two "spinless" nucleons with mass M and momentum k in the center-of-mass system. The elastic partial wave amplitude can be written [as a function of the scalar $s=4(k^2+M^2)$][11] as

$$A_l(s)=(1/2i\rho_l)(\eta_l(s)e^{2i\delta_l(s)}-1)=B_l(s)+{}^RA_l(s)\,, \quad (1)$$

where $\rho_l(s)$ is a kinematical factor and $B_l(s)$ is regular in the physical region, whereas ${}^RA_l(s)$ only has cuts for $s>4M^2\equiv s_E$. The inelastic partial wave cross section $\sigma_r{}^l$ is determined by η_l alone:

$$\sigma_r{}^l=\pi k^2(2l+1)[1-\eta_l{}^2(s)]\,. \quad (2)$$

Given the generalized potential $B_l(s)$, which contains the left-hand discontinuity, unitarity determines the right-hand discontinuity in $A_l(s)$:

$$A_l(s)=B_l(s)+\frac{1}{\pi}\int_{s_E}^{\infty}\frac{ds'}{(s'-s-i\epsilon)}|A_l(s')|^2\rho_l(s') +\frac{1}{\pi}\int_{s_I}^{\infty}\frac{ds'}{(s'-s-i\epsilon)}\frac{1-\eta^2{}_l(s')}{4\rho_l(s')}\,, \quad (3)$$

where s_I is the inelastic threshold. The nonlinear integral equation, (3), determines the real part of the scattering phase shift $\delta_l(s)$.

Froissart[8] has given a method of reducing (3) to a form where it can be treated by the N/D formalism. However, it does not seem as convenient a generalization of the pure elastic $[\eta_l(s)=1]$ case[4,12] as that of Frye and Warnock.[7] The Frye and Warnock equations which we will use for our calculations can be obtained as follows:

Let

$$A_l(s)=N_l(s)/D_l(s) \quad (4)$$

where, as in the pure elastic case, we assume that $D_l(s)$ has no left-hand cut and write

$$D_l{}^*(s)/D_l(s)=e^{2i\delta_l(s)}\,,\quad s\geq s_E \quad (5)$$

so that

$$N_l(s)=[1/2i\rho_l(s)][\eta_l(s)D_l{}^*(s)-D_l(s)]\,. \quad (6)$$

From (6) the following relations are immediately found[13]:

$$\mathrm{Im}D_l(s)=-\frac{2\rho_l(s)}{1+\eta_l(s)}\,\mathrm{Re}N_l(s)\theta(s-s_E)\,, \quad (7)$$

$$\mathrm{Im}N_l(s)=\frac{1-\eta_l(s)}{2\rho_l(s)}\,\mathrm{Re}D_l(s)\,. \quad (8)$$

Thus, in addition to the "usual" left-hand cut, N_l has a right-hand cut starting at the lowest inelastic threshold.

Using (7), we write a (subtracted) dispersion relation for $D_l(s)$:

$$\mathrm{Re}D_l(s)=1-\frac{s-s_0}{\pi}P\int_{s_E}^{\infty}\frac{2\rho_l(s')\,\mathrm{Re}N_l(s')ds'}{[1+\eta_l(s')](s'-s_0)(s'-s)}\,. \quad (9)$$

Now rewrite (1) as

$$N_l(s)/D_l(s)=B_l(s)+\frac{1}{\pi}\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s-i\epsilon)}+{}^R\bar{A}_l(s)\,.$$

Thus the expression

$$C_l(s)\equiv N_l(s)-B_l(s)D_l(s) -\frac{1}{\pi}\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s-i\epsilon)}D_l(s) \quad (10)$$

has *only* a right-hand cut beginning at s_E. From (7) and (8), we obtain for $s>s_E$

$$\mathrm{Re}C_l(s)=\frac{2\eta_l(s)}{1+\eta_l(s)}\,\mathrm{Re}N_l(s) -\left(B_l(s)+\frac{P}{\pi}\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s)}\right)\times\mathrm{Re}D_l(s)\,, \quad (11)$$

$$\mathrm{Im}C_l(s)=\left(B_l(s)+\frac{P}{\pi}\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s)}\right)\times\frac{2\rho_l(s)}{1+\eta_l(s)}\,\mathrm{Re}N_l(s)\,. \quad (12)$$

[9] L. Soroko, Zh. Eksperim. i Teor. Fiz. **35**, 276 (1958) [English transl.: Soviet Phys.—JETP 8, 190 (1959)].

[10] The quantitative behavior of δ depends on the assumed high-energy behavior of η: appropriate choices can be made so that $\delta(E)$ agrees with the nonunique phase shift analyses at 650 and 970 MeV (Ref. 2 and 3).

[11] We use units $\hbar=c=1$.

[12] J. Uretsky, Phys. Rev. **123**, 1459 (1961).

[13] J. Ball and W. Frazer, Phys. Rev. Letters **7**, 204 (1961).

Finally, writing a dispersion relation for $C_l(s)$ using (11) and (12), and (9) to express $\mathrm{Re}D_l(s)$ in terms $\mathrm{Re}N_l(s)$, we have

$$\frac{2\eta_l(s)}{1+\eta_l(s)}N_l(s)=\bar{B}_l(s)+\int_{s_E}^{\infty}\frac{2\rho_l(s')\,\mathrm{Re}N_l(s')ds'}{[1+\eta_l(s')](s'-s)}\left[\bar{B}_l(s')-\left(\frac{s-s_0}{s'-s_0}\right)\bar{B}_l(s)\right], \quad (13)$$

where

$$\bar{B}_l(s)=B_l(s)+\frac{P}{\pi}\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s)}. \quad (14)$$

These equations are a straightforward extension of the "pure elastic" N/D equations. With "proper" behavior of $B_l(s)$ and $\eta_l(s)$ at $s=\infty$,[14] the integral equation (13) for $N_l(s)$ is readily solved numerically by the matrix inversion method. The asymptotic and threshold behavior of (13) are discussed in the following section.

III. CALCULATIONS AND DISCUSSION

We encounter the same difficulties that occur in obtaining the correct threshold behavior as for the pure elastic $[\eta_l(s)=1]$ case: approximating $B_l(s)$ by the partial wave projection of single-particle exchange contributions, the correct threshold behavior of the unitarized amplitude for $l>0$ must be forced. One subtraction can be performed in the integral equation for N and still retain an acceptable behavior for large s. For $l>0$, we use the procedure of Scotti and Wong: make one subtraction at threshold in the equation for N, (13), and introduce an $(l-1)$ order pole on the left-hand cut, i.e., we choose $\rho_l(s)$ to have the form

$$\rho_l(s)=M\left(\frac{s-4M^2}{s-s_1}\right)^{l-1}\left(\frac{s-4M^2}{s}\right)^{1/2}. \quad (15)$$

With the above $\rho_l(s)$, the second term in (14) will diverge if $\eta_l(\infty)\neq 1$. Two points of view might be adopted: (1) The contribution to $1-\eta_l$ at large s from an individual inelastic channel goes to zero so that, if we consider a finite number of inelastic channels, $\eta_l(\infty)=1$. (ii) However, physically we expect that as the energy becomes infinite, an infinite number of inelastic channels open up and that whereas $\delta_l\to 0$, $\eta_l\to\eta_l(\infty)<1$;

$$A_l(s)\underset{s\to\infty}{\longrightarrow}\frac{\eta_l(\infty)-1}{2i\rho(\infty)}. \quad (16)$$

Clearly, far away left-hand singularities for this infinite number of inelastic channels must be important in determining the asymptotic behavior (16) of the amplitude. If, as indicated from the Regge description of high-energy scattering, $A_l(s)$ approaches the same limit (16) as $s\to\pm\infty$,[15] then the $B_l(s)$ term of (1) contains a term of the form

$$-\frac{1}{\pi}\int_{s_L}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'+s)}$$

for sufficiently large s_L. We will *assume* that $B_l(s)$ may be expressed as

$$B_l(s)=\mathbf{B}_l(s)-\frac{1}{\pi}\int_{s_L}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'+s)}, \quad (17)$$

where $\mathbf{B}_l(s)$ is the Born term calculated by S-W. Then for $l>1$, the subtracted (at $s=s_E$) form of (13) for N_l has a $\bar{B}_l(s)$, (14), given by

$$\bar{B}_l(s)=\mathbf{B}_l(s)+\frac{s-s_E}{\pi}\left[P\int_{s_I}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'-s)(s'-s_E)}+\int_{s_L}^{\infty}\frac{[1-\eta_l(s')]ds'}{2\rho_l(s')(s'+s)(s'+s_E)}\right]. \quad (18)$$

We see then that $\bar{B}_l(s)$ is bounded as $s\to\infty$ for any form for $\eta_l(s)$ if $B_l(s)$ is also bounded. It follows then that (13) is a Fredholm integral equation which may be solved for $\mathrm{Re}N_l(s)$ provided that $\eta_l(s)\neq 0$.[16]

TABLE I. Left-hand-cut Born term for the $^1D_2$ partial wave as determined by S-W (with $m_\pi=1$). The S-W ρ was of the form $(s-4M^2/s)^{1/2}M$.

k^2	$\mathbf{B}_{\text{S-W}}(s)$	$d\mathbf{B}_{\text{S-W}}(s)/ds$
0.009691	0.000026	0.001270
0.089222	0.001415	0.006351
0.246947	0.006061	0.007382
0.491235	0.012564	0.006008
0.819426	0.019497	0.004697
1.23814	0.026591	0.003875
1.75351	0.033987	0.003348
2.37329	0.041743	0.002928
3.10729	0.049737	0.002528
3.96771	0.057723	0.002121
4.96972	0.065384	0.001714
6.13224	0.072407	0.001320
7.47888	0.078496	0.000954
9.03936	0.083389	0.000627
10.8514	0.086864	0.000344
12.9636	0.088734	0.000109
15.4392	0.088849	−0.000079
18.3618	0.087098	−0.000213
21.5447	0.083439	−0.000303
26.0448	0.077961	−0.000342
31.1843	0.071013	−0.000327
37.5919	0.063449	−0.000256
45.7787	0.057101	−0.000128
56.5343	0.055526	0.000052
71.2880	0.064878	0.000252
92.6369	0.093683	0.000393
126.420	0.147304	0.000369
187.284	0.211428	0.000164
329.523	0.241625	−0.000003
1041.35	0.191039	−0.000013

[14] See Ref. 7 for a general discussion.

[15] R. Omnes, Phys. Rev. **133**, B1543 (1964).

[16] η can approach 0 as $s\to\infty$ provided $\bar{B}_l(s)\to 0$ faster (see Ref. 7). For our problem, $\eta(s)\to_{s\to\infty}(\ln s)^{-1}$ was acceptable behavior.

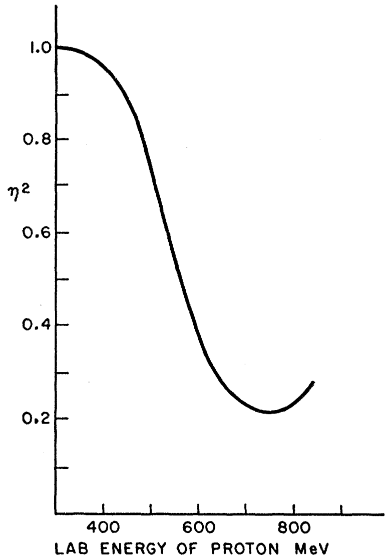

FIG. 1. Plot of η^2 for $^1D_2$ partial wave taken from Ref. 9.

The calculations presented in this paper were restricted to the $^1D_2$ partial wave, since η for *this* partial wave is known experimentally up to $E_L \sim 800$ MeV and $1-\eta$ is large.

The Born term $\mathbf{B}_l(s)$ which was used in the present calculations was taken to be that determined by S–W. It was found by considering the exchange of a pion, an $I=0$ s-wave π–π system and the multipion resonances η, ρ, ϕ, ω, in nucleon-antinucleon scattering. This is then related, using crossing symmetry, to N–N scattering and the $^1D_2$ partial wave is projected out. For the vector particle (ρ,ϕ,ω) exchanges, the logarithmic divergence in $\mathbf{B}_l(s)$ for $s\to\infty$ was avoided by a cutoff procedure which is related to the interpretation of these particles as Regge poles. Instead of giving the input parameters in

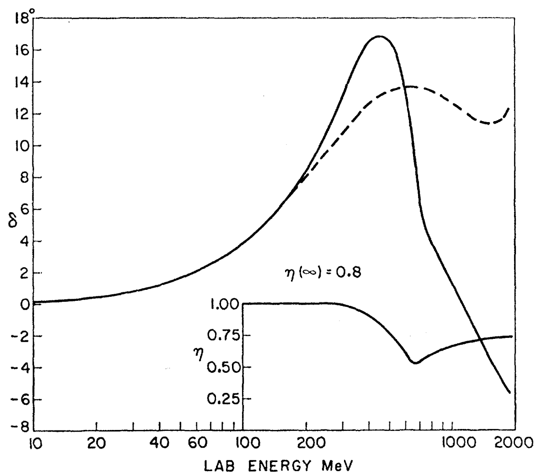

FIG. 2. Plot of (real part of) the phase shift δ for $^1D_2$ partial wave as a function of the laboratory kinetic energy E_L. The dashed curve corresponds to the pure elastic case [i.e., $\eta(s)\equiv 1$]. The asymptotic behavior of η is of the form $As/(B+s)$ where A and B are constants.

terms of the relevant coupling constants and Regge slopes, we present in Table I values of $\mathbf{B}_l(s)$ and $d\mathbf{B}_l(s)/ds$ for the $^1D_2$ partial wave determined by S–W and used in our calculations.

Mandelstam[17] calculated cross sections for $N+N\to N+N+\pi$ by assuming that the pion production occurs in only a few angular momentum states, and that the transition matrix element is constant except for factors due to the final state N–N and π–N interactions. The outgoing π is assumed to be in $J=\frac{3}{2}$, $I=\frac{3}{2}$ resonant state with one of the nucleons. A three-parameter theory was constructed which gave a good representation of both total and differential pion production cross sections up to $\sim$700 MeV except near threshold where nonresonant production is important and further parameters were needed.

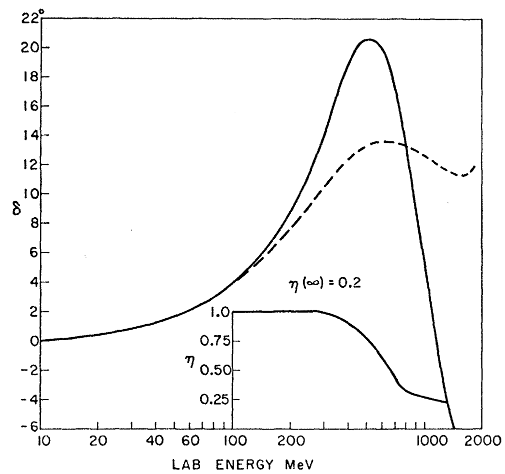

FIG. 3. Same as Fig. 2.

Employing Mandelstam's theory, Soroko[9] used experimental data for the reactions listed below to compute $\eta(s)$ for the $^1D_2$ partial wave in p–p scattering. The important channels for protons with $E_L \lesssim 800$ MeV are

$$\begin{aligned}
&(a)\quad p+p\to p+p\,,\\
&(b)\quad \to d+\pi^+ \quad ({}^3s_1p)_2\,,\\
&(c)\quad \to p+n+\pi^+ \quad ({}^3s_1p)_2\,,\\
&(d)\quad \to p+p+\pi^0 \quad ({}^3p_2s)_2\,,
\end{aligned}$$

where, e.g., $({}^3s_1p)_2$ means that the deuteron (or n–p) is in a $^3s_1$ state and the π^+ is in a p state relative to the d (or the n–p center of mass). The last reaction is negligible in comparison to the first three. Soroko's results, reproduced graphically in Fig. 1, were used in our calculation of the real part of the phase shift $\delta(s)$ for the $^1D_2$ partial wave.

In our calculations we represented $\eta(s)$ by various analytic forms. They all had the features of behaving

[17] S. Mandelstam, Proc. Roy. Soc. (London) **A244**, 491 (1958).

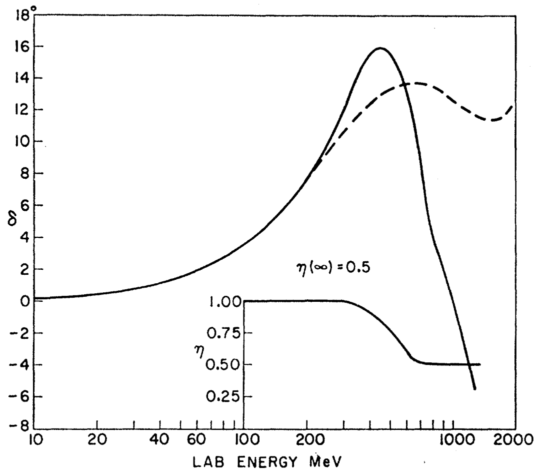

FIG. 4. Same as Fig. 2.

like

$$1-\eta(s) \propto q^3$$

right above the π^+d threshold (where q is the momentum of this channel in the center-of-mass system) and giving a good fit to Soroko's data up to $E_L=700$ MeV. For higher energies $\eta(s)$ was allowed to assume different asymptotic forms and approach different limits $\eta(s=\infty)$. The calculated $\delta(s)$ for various choices of the high-energy behavior of $\eta(s)$ are given in Figs. 2–7. For comparison the values of δ for $\eta(s)=1$ (i.e., no inelastic scattering) are given by the dashed curves. The sensitivity of the results to the choices of the parameters s_1 [see Eq. (15)] and s_L [see Eq. (17)] were tested and found to be small.[18]

We observe the general feature that including the inelastic terms acts as an attraction below the inelastic threshold. However, we note that there is very little

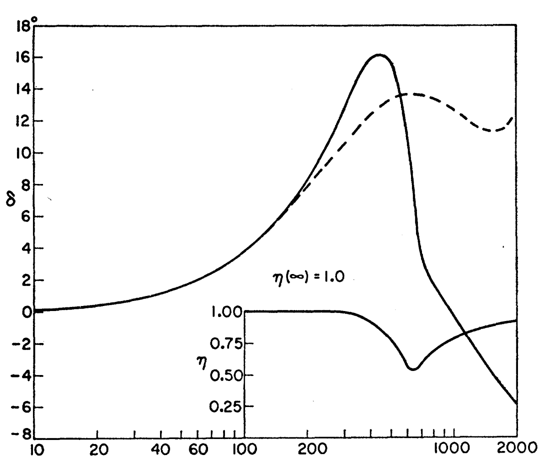

FIG. 5. Same as Fig. 2.

[18] Variations in s_1 from 0 to 100 m_π^2 and s_L from 1000 to 5000 m_π^2 each yielded changes in $\delta(E_L=200\text{ MeV})$ of $\lesssim 0.5°$. Changes for higher E_L were somewhat larger but the results were qualitatively the same.

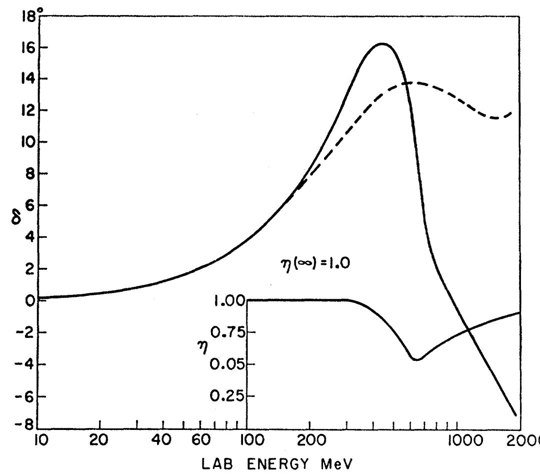

FIG. 6. Same as Fig. 2 except that the asymptotic behavior of η is of the form $[1+A/\ln(s/B)]$.

difference from the "pure elastic" calculations up to $E_L \approx 200$ MeV. The calculated δ peak at $\approx$400–500 MeV and go negative at $\gtrsim 1$ BeV.[10]

Although we restricted our present calculations to the ${}^1D_2$ partial wave, we would like to emphasize the following features which should apply to the *N–N* problem in general.

(i) Despite the fact that $1-\eta_l$ is quite small in the energy region 300–500 MeV, any quantitative *dynamical* calculation for $\delta_l(s)$ in this energy region (or higher) must include the effect of the inelastic cut.

(ii) Since the present dynamical calculations indicate δ is rapidly varying in energy above 300 MeV, it may be misleading to do an energy average of the experimental data in order to perform a phase shift analysis.

(iii) Finally, we want to stress the importance of experimentally extending our knowledge of the η's to higher energy: quantitative dynamical calculations of

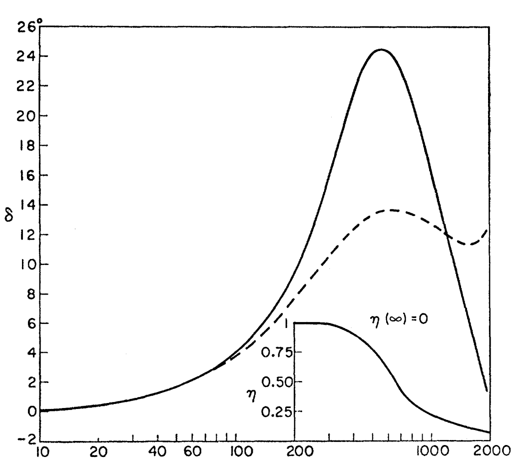

FIG. 7. Same as Fig. 2 except that the asymptotic behavior of η is of the form $A/\ln(s/B)$.

the δ's above 300 MeV depend crucially on them. This can be attacked in two ways: at moderate energies detailed knowledge of the inelastic production angular distribution might be combined with analyses similar to those of Mandelstam and Soroko. At high energies the δ's probably go to zero so that a phase shift analysis of the elastic scattering might be done solely in terms of the η's.

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Photon as a Symmetry-Breaking Solution to Field Theory. I*†

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The mechanism which guarantees the consistency of the angular-momentum conservation and commutation rules of a Lorentz-invariant theory with the requirement that the vacuum expectation of a vector operator be nonvanishing is examined in detail. A theory originally proposed by Bjorken which reproduces ordinary electrodynamics is presented in a manner which allows the calculation of the parameters of the theory. In particular the "consistency condition" is displayed and found to be quadratically, not cubically, divergent. It is shown that the original Bjorken solution occurs when the cutoff condition of the theory is taken literally. This attitude results in difficulties with current conservation and leads to transitions between the standard vacuum and anomalous degenerate states. These transitions alone, and not the ones directly involving the massless vector particles induced by the broken symmetry, are responsible for the ultimate consistency of the theory. An alternative formulation of the theory which does not take the cutoff so seriously, and hence places emphasis on the underlying operator structure rather than the perturbation Green's functions of the theory, is proposed. This presentation is essentially equivalent to the original formulation since it differs only by gauge terms. However, in this case no difficulty is encountered with current conservation and the theory is consistent in the manner required by normal formulations of the Goldstone theorem.

INTRODUCTION

In the last few years increasing amounts of evidence have been gathered to indicate the quantum field theory has sufficient untapped potential to allow it to deal fairly simply with the vast number of observed particles. The basis of this evidence is the observation that it is probably possible for one field operator to be associated with more than one particle. This possibility was first explored by Heisenberg[1,2] and his co-workers in a series of papers on nonlinear field theory. However, Heisenberg's work involved the introduction of several concepts which are radically different from those of ordinary field theory.

Using less radical concepts developed recently to explain the phenomena of superconductivity,[3,4] it was possible for Jona-Lasinio and Nambu[5,6] to develop a nonlinear theory in which the "pion" is not introduced as a separate field but as an excitation associated with a current of the fermion field. The basic assumption that allows the occurence of the pion is that the vacuum is a degenerate state. In particular, it is assumed that the vacuum is no longer invariant under the continuous group of rotations in γ_5 space. It is then said that the γ_5 symmetry is "broken." This assumption, although it is alien to long cherished beliefs in the quantum field theory of particles, is not at all unusual in other branches of physics. The ground state of a superconductor or ordinary paramagnetism are common examples of "broken symmetries."

In this work we shall study a way of "creating" a photon by "breaking" the invariance of the vacuum under Lorentz transformations. The suggestion that a photon might be created in this manner was first made by Bjorken.[7] The possibility of generating a photon through a four-fermion interaction has also been previously examined by Heisenberg[1,2] and Birula.[8] The essential feature in the Bjorken-type theory is that the masslessness of the derived particle is associated with

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